Chapter 2

PROBLEMS: 12, 13, 16, 19, 20, 23, 35, 51, 55, 64, 67, 86, 95

12 •• Which of the position-versus-time curves in Figure 2-28 best shows the motion of an object (*a*) with positive acceleration, (*b*) with constant positive velocity, (*c*) that is always at rest, and (*d*) with negative acceleration? (There may be more than one correct answer for each part of the problem.)

Determine the Concept The slope of an $x(t)$ curve at any point in time represents the speed at that instant. The way the slope changes as time increases gives the sign of the acceleration. If the slope becomes less negative or more positive as time increases (as you move to the right on the time axis), then the acceleration is positive. If the slope becomes less positive or more negative, then the acceleration is negative. The slope of the slope of an $x(t)$ curve at any point in time represents the acceleration at that instant.

(*a*) The correct answer is $\boxed{(d)}$. The slope of curve (*d*) is positive and increasing. Therefore the velocity and acceleration are positive. We need more information to conclude that *a* is constant.

(*b*) The correct answer is $\boxed{(b)}$. The slope of curve (*b*) is positive and constant. Therefore the velocity is positive and constant.

(*c*) The correct answer is $\boxed{(e)}$. The slope of curve (*e*) is zero. Therefore, the velocity and acceleration are zero.

(*d*) The correct answers are $\boxed{(a) \text{ and } (c)}$. The slope of curve (*a*) is negative and becomes more negative as time increases. Therefore the velocity is negative and the acceleration is negative. The slope of curve (*c*) is positive and decreasing. Therefore the velocity is positive and the acceleration is negative.

13 •• **[SSM]** Which of the velocity-versus-time curves in figure 2-29 best describes the motion of an object (*a*) with constant positive acceleration, (*b*) with positive acceleration that is decreasing with time, (*c*) with positive acceleration that is increasing with time, and (*d*) with no acceleration? (There may be more than one correct answer for each part of the problem.)

Determine the Concept The slope of a $v(t)$ curve at any point in time represents the acceleration at that instant.

(*a*) The correct answer is $\boxed{(b)}$. The slope of curve (*b*) is constant and positive. Therefore the acceleration is constant and positive.

(*b*) The correct answer is $\boxed{(c)}$. The slope of curve (*c*) is positive and decreasing with time. Therefore the acceleration is positive and decreasing with time.
(*c*) The correct answer is $\boxed{(d)}$. The slope of curve (*d*) is positive and increasing with time. Therefore the acceleration is positive and increasing with time.
(*d*) The correct answer is $\boxed{(e)}$. The slope of curve (*e*) is zero. Therefore the velocity is constant and the acceleration is zero.

16 •• For each of the four graphs of x versus t in Figure 2-31 answer the following questions. (*a*) Is the velocity at time t_2 greater than, less than, or equal to the velocity at time t_1? (*b*) Is the speed at time t_2 greater than, less than, or equal to the speed at time t_1?

Determine the Concept In one-dimensional motion, the velocity is the slope of a position-versus-time plot and can be either positive or negative. On the other hand, the speed is the magnitude of the velocity and can only be positive. We'll use v to denote velocity and the word ″speed″ for how fast the object is moving.

(*a*)	(*b*)
curve a: $v(t_2) < v(t_1)$	curve a: $\text{speed}(t_2) < \text{speed}(t_1)$
curve b: $v(t_2) = v(t_1)$	curve b: $\text{speed}(t_2) = \text{speed}(t_1)$
curve c: $v(t_2) > v(t_1)$	curve c: $\text{speed}(t_2) < \text{speed}(t_1)$
curve d: $v(t_2) < v(t_1)$	curve d: $\text{speed}(t_2) > \text{speed}(t_1)$

19 •• **[SSM]** A ball is thrown straight up. Neglect any effects due to air resistance. (*a*) What is the velocity of the ball at the top of its flight? (*b*) What is its acceleration at that point? (*c*) What is different about the velocity and acceleration at the top of the flight if instead the ball impacts a horizontal ceiling very hard and then returns.

Determine the Concept In the absence of air resistance, the ball will experience a constant acceleration. In the graph that follows, a coordinate system was chosen in which the origin is at the point of release and the upward direction is positive. The graph shows the velocity of a ball that has been thrown straight upward with an initial speed of 30 m/s as a function of time.

(*a*) $\boldsymbol{v}_{\text{top of flight}} = \boxed{0}$
(*b*) Note that the acceleration of the ball is the same at every point of its trajectory, including the point at which $v = 0$ (at the top of its flight). Hence $\boldsymbol{a}_{\text{top of flight}} = \boxed{-\boldsymbol{g}}$
(c) If the ball impacts a horizontal ceiling very hard and then returns, its velocity at the top of its flight is still zero and its acceleration is still downward but greater than g in magnitude.

20 •• An object that is launched straight up from the ground, reaches a maximum height H, and falls straight back down to the ground, hitting it T seconds after launch. Neglect any effects due to air resistance. (*a*) Express the average speed for the entire trip as a function of H and T. (*b*) Express the average speed for the same interval of time as a function of the initial launch speed v_0.

Picture the Problem The average speed is being requested as opposed to average velocity. We can use the definition of average speed as distance traveled divided by the elapsed time and expression for the average speed of an object when it is experiencing constant acceleration to express v_{av} in terms of v_0.

(*a*) The average speed is defined as the total distance traveled divided by the change in time:	$\boldsymbol{v}_{av} = \dfrac{\text{total distance traveled}}{\text{total time}}$
Substitute for the total distance traveled and the total time and simplify to obtain:	$v_{av} = \dfrac{H+H}{T} = \boxed{\dfrac{2H}{T}}$
(*b*) Express the average speed for the upward flight of the object:	$v_{av,up} = \dfrac{v_0 + 0}{2} = \dfrac{H}{\frac{1}{2}T} \Rightarrow H = \frac{1}{4}v_0T$
Express the average speed for the downward flight of the object:	$v_{av,down} = \dfrac{0 + v_0}{2} = \dfrac{H}{\frac{1}{2}T} \Rightarrow H = \frac{1}{4}v_0T$
Substitute in our expression for v_{av} in (*a*) and simplify to obtain:	$v_{av} = \dfrac{\frac{1}{4}v_0T + \frac{1}{4}v_0T}{T} = \boxed{\frac{1}{2}v_0}$ Because $v_0 \neq 0$, the average speed is not zero.

Remarks: 1) Because this motion involves a roundtrip, if the question asked for ″average velocity″, the answer would be zero. 2) Another easy way to obtain this result is take the absolute value of the velocity of the object to obtain a graph of its speed as a function of time. A simple geometric argument leads to the result we obtained above.

23 •• You are driving a Porsche that accelerates uniformly from 80.5 km/h (50 mi/h) at t = 0.00 to 113 km/h (70 mi/h) at t = 9.00 s. (*a*) Which graph in Figure 2-32 best describes the velocity of your car? (*b*) Sketch a position-versus- time graph showing the location of your car during these nine seconds, assuming we let its position x be zero at t = 0.

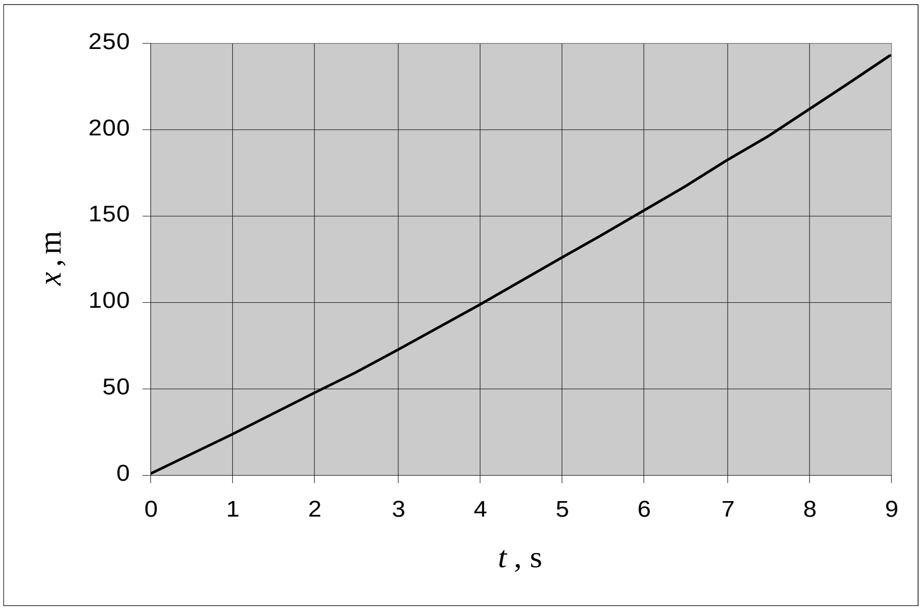
Determine the Concept Because the Porsche accelerates uniformly, we need to look for a graph that represents constant acceleration.

(*a*) Because the Porsche has a constant acceleration that is positive (the velocity is increasing), we must look for a velocity-versus-time curve with a positive constant slope and a nonzero intercept. Such a graph is shown in $\boxed{(c)}$.

(*b*) Use the data given in the problem statement to determine that the acceleration of the Porsche is 1.00 m/s^2 and that its initial speed is 22.4 m/s. The equation describing the position of the car as a function of time is

$$x = \left(22.4\ \text{m/s}^2\right)t + \tfrac{1}{2}\left(1.00\ \text{m/s}^2\right)t^2.$$

The following graph of this equation was plotted using a spreadsheet program.



35 •• Figure 2-37 shows nine graphs of position, velocity, and acceleration for objects in motion along a straight line. Indicate the graphs that meet the following conditions: (*a*) Velocity is constant, (*b*) velocity reverses its direction, (*c*) acceleration is constant, and (*d*) acceleration is not constant. (*e*) Which graphs of position, velocity, and acceleration are mutually consistent?

Determine the Concept Velocity is the slope and acceleration is the slope of the slope of a position-versus-time curve. Acceleration is the slope of a velocity-versus-time curve.

(*a*) Graphs $\boxed{(a), (f), \text{and } (i)}$ describe motion at constant velocity. For constant velocity, *x* versus *t* must be a straight line; *v*-versus-*t* must be a horizontal straight line; and *a* versus *t* must be a straight horizontal line at *a* = 0.

(*b*) Graphs $\boxed{(c) \text{ and } (d)}$ describe motion in which the velocity reverses its

direction. For velocity to reverse its direction x-versus-t must have a slope that changes sign and v versus t must cross the time axis. The acceleration cannot remain zero at all times.
(*c*) Graphs **(a), (d), (e), (f), (h), and (i)** describe motion with constant acceleration. For constant acceleration, x versus t must be a straight line or a parabola, v versus t must be a straight line, and a versus t must be a horizontal straight line.
(*d*) Graphs **(b), (c), and (g)** describe motion with non-constant acceleration. For non-constant acceleration, x versus t must not be a straight line or a parabola; v versus t must not be a straight line, or a versus t must not be a horizontal straight line.
(*e*) The following pairs of graphs are mutually consistent: **(a) and (i)**, **(d) and (h)**, and **(f) and (i)**. For two graphs to be mutually consistent, the curves must be consistent with the definitions of velocity and acceleration.

51 •• Figure 2-39 shows the position of a particle as a function of time. Find the average velocities for the time intervals *a*, *b*, *c*, and *d* indicated in the figure.

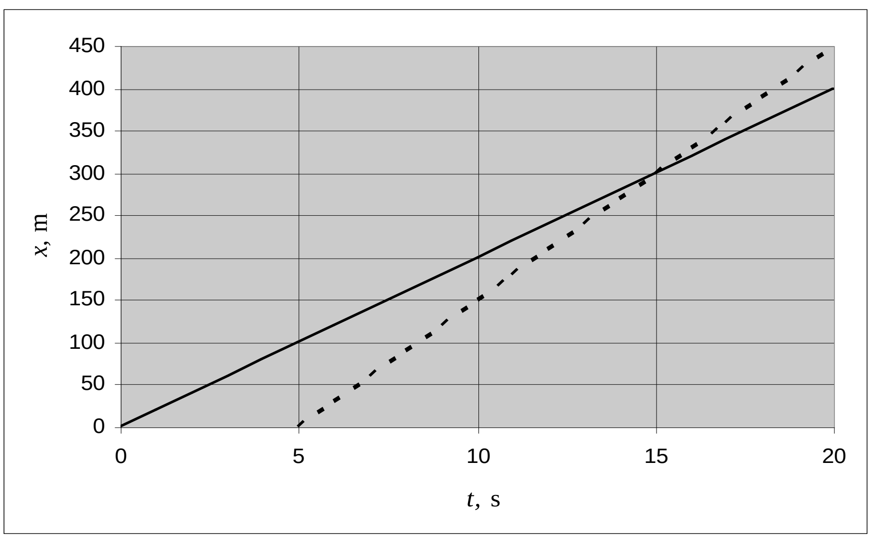
Picture the Problem The average velocity in a time interval is defined as the displacement divided by the time elapsed; that is $v_{av} = \Delta x/\Delta t$.

(*a*) $\Delta x_a = 0$	$v_{av} = \boxed{0}$
(*b*) $\Delta x_b = 1$ m and $\Delta t_b = 3$ s	$v_{av} = \boxed{0.3\,\text{m/s}}$
(*c*) $\Delta x_c = -6$ m and $\Delta t_c = 3$ s	$v_{av} = \boxed{-2\,\text{m/s}}$
(*d*) $\Delta x_d = 3$ m and $\Delta t_d = 3$ s	$v_{av} = \boxed{1\,\text{m/s}}$

55 •• **[SSM]** A car traveling at a constant speed of 20 m/s passes an intersection at time $t = 0$. A second car traveling at a constant speed of 30 m/s in the same direction passes the same intersection 5.0 s later. (*a*) Sketch the position functions $x_1(t)$ and $x_2(t)$ for the two cars for the interval $0 \le t \le 20$ s. (*b*) Determine when the second car will overtake the first. (*c*) How far from the intersection will the two cars be when they pull even? (*d*) Where is the first car when the second car passes the intersection?

Picture the Problem One way to solve this problem is by using a graphing calculator to plot the positions of each car as a function of time. Plotting these

positions as functions of time allows us to visualize the motion of the two cars relative to the (fixed) ground. More importantly, it allows us to see the motion of the two cars relative to each other. We can, for example, tell how far apart the cars are at any given time by determining the length of a vertical line segment from one curve to the other.

(*a*) Letting the origin of our coordinate system be at the intersection, the position of the slower car, $x_1(t)$, is given by:	$x_1(t) = 20t$ where x_1 is in meters if t is in seconds.
Because the faster car is also moving at a constant speed, we know that the position of this car is given by a function of the form:	$x_2(t) = 30t + b$
We know that when $t = 5.0$ s, this second car is at the intersection (that is, $x_2(5.0\text{ s}) = 0$). Using this information, you can convince yourself that:	$b = -150$ m
Thus, the position of the faster car is given by:	$x_2(t) = 30t - 150$
One can use a graphing calculator, graphing paper, or a spreadsheet to obtain the following graphs of $x_1(t)$ (the solid line) and $x_2(t)$ (the dashed line):	
	
(*b*) Use the time coordinate of the intersection of the two lines to determine the time at which the second car overtakes the first:	From the intersection of the two lines, one can see that the second car will "overtake" (catch up to) the first car at $\boxed{t = 15\text{ s}}$.

(*c*) Use the position coordinate of the intersection of the two lines to determine the distance from the intersection at which the second car catches up to the first car:	From the intersection of the two lines, one can see that the distance from the intersection is $\boxed{300\text{ m.}}$
(*d*) Draw a vertical line from $t = 5$ s to the solid line and then read the position coordinate of the intersection of the vertical line and the solid line to determine the position of the first car when the second car went through the intersection. From the graph, when the second car passes the intersection, the first car was $\boxed{100\text{ m ahead.}}$	

64 • An object projected vertically upward with initial speed v_0 attains a maximum height h above its launch point. Another object projected up with initial speed $2v_0$ from the same height will attain a maximum height of (*a*) 4*h*, (*b*) 3*h*,
(*c*) 2*h*, (*d*) *h*. (Air resistance is negligible.)

Picture the Problem Because the acceleration is constant ($-g$) we can use a constant-acceleration equation to find the height of the projectile.

Using a constant-acceleration equation, express the height of the object as a function of its initial speed, the acceleration due to gravity, and its displacement:	$v^2 = v_0^2 - 2g\Delta y$ or, because $v(h) = 0$, $0 = v_0^2 - 2gh \Rightarrow h = \dfrac{v_0^2}{2g}$
Express the ratio of the maximum height of the second object to that of the first object and simplify to obtain:	$\dfrac{h_{\text{2nd object}}}{h_{\text{1st object}}} = \dfrac{\dfrac{(2v_0)^2}{2g}}{\dfrac{(v_0)^2}{2g}} = 4$
Solving for $h_{\text{2nd object}}$ yields:	$h_{\text{2nd object}} = 4h \Rightarrow (a)$ is correct.

67 • **[SSM]** An object traveling along the x axis at constant acceleration has a velocity of +10 m/s when it is at $x = 6.0$ m and of +15 m/s when it is at
$x = 10$ m. What is its acceleration?

Picture the Problem Because the acceleration of the object is constant we can use constant-acceleration equations to describe its motion.

Using a constant-acceleration equation, relate the velocity to the acceleration and the displacement:	$v^2 = v_0^2 + 2a\,\Delta x \Rightarrow a = \dfrac{v^2 - v_0^2}{2\,\Delta x}$
Substitute numerical values and evaluate *a*:	$\boldsymbol{a} = \dfrac{(15^2 - 10^2)\,\text{m}^2/\text{s}^2}{2(10\text{ m} - 6.0\text{ m})} = \boxed{16\text{ m/s}^2}$

86 •• A motorcycle officer hidden at an intersection observes a car driven by an oblivious driver who ignores a stop sign and continues through the intersection at constant speed. The police officer takes off in pursuit 2.0 s after the car has passed the stop sign. She accelerates at 4.2 m/s^2 until her speed is 110 km/h, and then continues at this speed until she catches the car. At that instant, the car is 1.4 km from the intersection. (*a*) How long did it take for the officer to catch up to the car? (*b*) How fast was the car traveling?

Picture the Problem The acceleration of the police officer's car is positive and constant and the acceleration of the speeder's car is zero. Choose a coordinate system such that the direction of motion of the two vehicles is the positive direction and the origin is at the stop sign.

(*a*) The time traveled by the car is given by:	$t_{car} = 2.0\,\text{s} + t_1 + t_2$ (1) where t_1 is the time during which the motorcycle was accelerating and t_2 is the time during which the motorcycle moved with constant speed.
Convert 110 km/h into m/s:	$v_1 = 110\,\frac{\text{km}}{\text{h}} \times \frac{10^3\,\text{m}}{\text{km}} \times \frac{1\,\text{h}}{3600\,\text{s}} = 30.56\,\frac{\text{m}}{\text{s}}$
Express and evaluate t_1:	$t_1 = \frac{v_1}{a_{motorcycle}} = \frac{30.56\,\text{m/s}}{4.2\,\text{m/s}^2} = 7.276\,\text{s}$
Express and evaluate d_1:	$d_1 = \frac{1}{2}v_1t_1 = \frac{1}{2}(30.56\,\text{m/s})(7.276\,\text{s}) = 111.2\,\text{m}$
Determine d_2:	$d_2 = d_{caught} - d_1 = 1400\,\text{m} - 111\,\text{m} = 1289\,\text{m}$
Express and evaluate t_2:	$t_2 = \frac{d_2}{v_1} = \frac{1289\,\text{m}}{30.56\,\text{m/s}} = 42.18\,\text{s}$
Substitute in equation (1) and evaluate t_{car}:	$t_{car} = 2.0\,\text{s} + 7.3\,\text{s} + 42.2\,\text{s} = 51.5\,\text{s} = \boxed{52\,\text{s}}$
(*b*) The speed of the car when it was overtaken is the ratio of the distance it traveled to the elapsed time:	$v_{car} = \frac{d_{caught}}{t_{car}}$
Substitute numerical values and evaluate v_{car}:	$v_{car} = \left(\frac{1400\,\text{m}}{51.5\,\text{s}}\right)\left(\frac{1\,\text{mi/h}}{0.447\,\text{m/s}}\right) = \boxed{61\,\text{mi/h}}$

95 •• A train pulls away from a station with a constant acceleration of $0.40\ \text{m/s}^2$. A passenger arrives at a point next to the track 6.0 s after the end of the train has passed the very same point. What is the slowest constant speed at which she can run and still catch the train? On a single graph plot the position versus time curves for both the train and the passenger.

Picture the Problem Because the acceleration is constant, we can describe the motions of the train using constant-acceleration equations. Find expressions for the distances traveled, separately, by the train and the passenger. When are they equal? Note that the train is accelerating and the passenger runs at a constant minimum velocity (zero acceleration) such that she can just catch the train.

1. Using the subscripts ″train″ and ″p″ to refer to the train and the passenger and the subscript ″c″ to identify ″critical″ conditions, express the position of the train and the passenger:	$x_{\text{train,c}}(t_{\text{c}}) = \frac{a_{\text{train}}}{2}t_{\text{c}}^2$ and $x_{\text{p,c}}(t_{\text{c}}) = v_{\text{p,c}}(t_{\text{c}} - \Delta t)$
Express the critical conditions that must be satisfied if the passenger is to catch the train:	$v_{\text{train,c}} = v_{\text{p,c}}$ and $x_{\text{train,c}} = x_{\text{p,c}}$
2. Express the train's average velocity.	$v_{\text{av}}(0 \text{ to } t_{\text{c}}) = \frac{0 + v_{\text{train,c}}}{2} = \frac{v_{\text{train,c}}}{2}$
3. Using the definition of average velocity, express v_{av} in terms of $x_{\text{p,c}}$ and t_{c}.	$v_{\text{av}} \equiv \frac{\Delta x}{\Delta t} = \frac{0 + x_{\text{p,c}}}{0 + t_{\text{c}}} = \frac{x_{\text{p,c}}}{t_{\text{c}}}$
4. Combine steps 2 and 3 and solve for $x_{\text{p,c}}$.	$x_{\text{p,c}} = \frac{v_{\text{train,c}}t_{\text{c}}}{2}$
5. Combine steps 1 and 4 and solve for t_{c}.	$v_{\text{p,c}}(t_{\text{c}} - \Delta t) = \frac{v_{\text{train,c}}t_{\text{c}}}{2}$ or $t_{\text{c}} - \Delta t = \frac{t_{\text{c}}}{2}$ and $t_{\text{c}} = 2\,\Delta t = 2\ (6\text{ s}) = 12\text{ s}$
6. Finally, combine steps 1 and 5 and solve for $v_{\text{train, c}}$.	$\boldsymbol{v}_{\text{p,c}} = \boldsymbol{v}_{\text{train,c}} = \boldsymbol{a}_{\text{train}}\boldsymbol{t}_{\text{c}} = (0.40\ \text{m/s}^2)(12\ \text{s})$ $= \boxed{4.8\ \text{m/s}}$

The following graph shows the location of both the passenger and the train as a function of time. The parabolic solid curve is the graph of $x_{\text{train}}(t)$ for the accelerating train. The straight dashed line is passenger's position $x_{\text{p}}(t)$ if she arrives at $\Delta t = 6.0$ s after the train departs. When the passenger catches the train, our graph shows that her speed and that of the train must be equal ($v_{\text{train,c}} = v_{\text{p,c}}$).

Do you see why?